



## Stanion C.E (Aided) Primary School

### Calculation Policy

The following calculation policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school.

#### Age stage expectations

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **however it is vital that pupils are taught according to the stage that they are currently working at**, working at a lower stage if necessary until they are secure enough to move on. Understanding should be deepened through the use of varied representations and contexts.

#### Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems.

The document builds on the interconnectedness of mathematics and outlines the progression for addition, subtraction, multiplication and division. It is our intention that addition and subtraction should be taught at the same time to ensure children are able to see the clear links between the operations and the inverse nature of them along with multiplication and division.

#### LEARNING OUTCOMES

EARLY YEARS:	Page 2
KEY STAGE ONE:	Pages 2 and 3
KEY STAGE TWO (LOWER):	Pages 4 and 5
KEY STAGE TWO (HIGHER):	Pages 6, 7 and 8

**By the end of the academic year, children should have achieved the learning outcomes listed for each year group.**

**RECEPTION (EYFS):**

- children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number
- using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer
- they solve problems, including doubling, halving and sharing

**YEAR ONE (Key Stage 1):**

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens
- given a number, identify one more and one less
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least
- read and write numbers from 1 to 20 in numerals and words
- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square - 9$
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

**YEAR TWO (Key Stage 1):**

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forwards and backwards
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use and = signs
- read and write numbers to at least 100 in numerals and in words
- use place value and number facts to solve problems
- solve problems with addition and subtraction, using concrete objects and pictorial representations
- recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using: materials, arrays, repeated addition, mental methods, and multiplication and division facts

**YEAR THREE (Lower Key Stage 2):**

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations
- read and write numbers up to 1000 in numerals and in words
- add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects

**YEAR FOUR (Lower Key Stage 2):**

- count in multiples of 6, 7, 9, 25 and 1000
- find 1000 more or less than a given number
- count backwards through zero to include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare numbers beyond 1000

- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value
- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as  $n$  objects are connected to  $m$  objects

**YEAR FIVE (Upper Key Stage 2):**

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- solve number problems and practical problems that involve all of the above
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

- recognise and use square numbers and cube numbers, and the notation for squared ( 2 ) and cubed ( 3 )
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

**YEAR SIX (Upper Key Stage 2):**

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number and practical problems that involve all of the above
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division

- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

### **Stages of Calculation**

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved and the children's experience.

Children should be familiar with different ways to represent calculations across all year groups. They should develop the ability to explain how their method works and why they have chosen their methods. They should be challenged to prove their answers are correct using different representations or methods.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Concrete and pictorial methods should be used with all children to deepen understanding and build confidence.

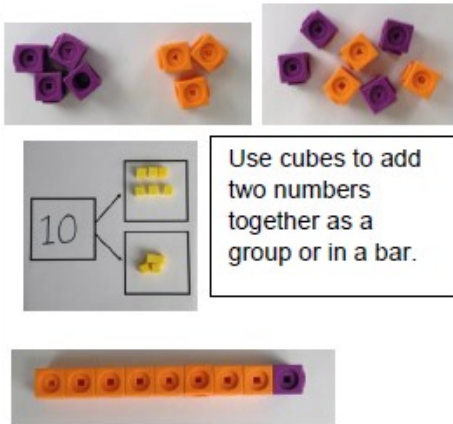
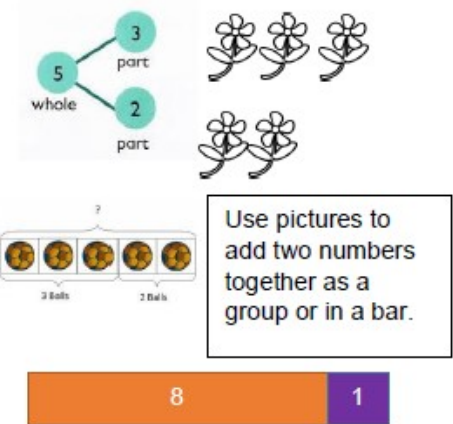


## Addition

### Progression of Addition

#### Stage 1

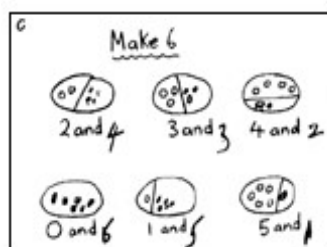
- Lots of practical activities and counting rhymes and songs;
- Teacher models written recording using objects, marks and pictures;
- Children begin to make own marks during structured play.

Concrete	Pictorial
 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>

#### Stage 2

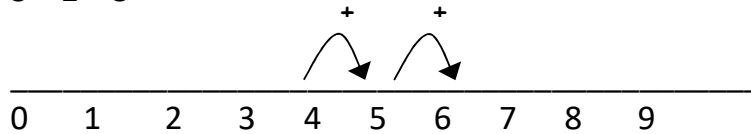
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation alongside practical experiences.

#### Example



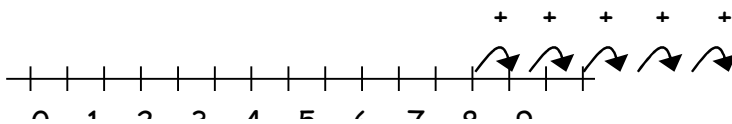
They use number lines and practical resources to support calculation and teacher demonstrate the use of number lines.

$$3 + 2 = 5$$

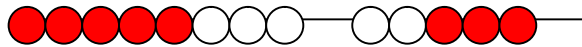


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



By the end of stage 2 children are selecting their own apparatus, using jottings (informal pencil and paper methods) and recording number sentences.

Children are encouraged to use a number square to add by counting on, initially in units, then in tens and units.

**Example 1**  $8 + 7 = 15$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

## Example 2


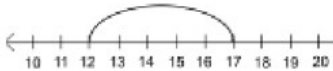
$$48+36=84$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

It is important to change the format of the square to that children become aware that the positions of numbers can change.

They will continue to use number lines, jottings and practical resources, selecting for themselves which best suits their needs or with support and guidance where required.




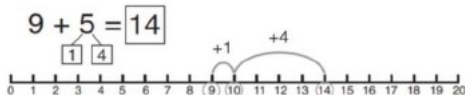
Concrete/Pictorial/Abstract:

 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	$12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
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### Stage 3

Children will use regrouping to ten to efficiently add numbers that go over 10.


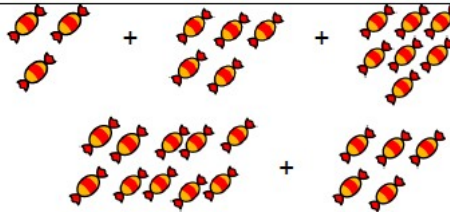
Concrete/Pictorial/Abstract:

 <p><math>6 + 5 = 11</math></p>  <p>Start with the bigger number and use the smaller number to make 10.</p>	 <p><math>3 + 9 =</math></p> <p><math>9 + 5 = 14</math></p>  <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p>	<p><math>7 + 4 = 11</math></p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
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### Stage 4

Children will explore adding three one-digit numbers using number bonds to 10 and regrouping to make 10.

Concrete/Pictorial/Abstract:

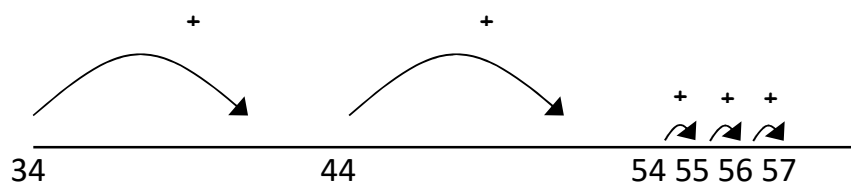
<p><math>4 + 7 + 6 = 17</math></p> <p>Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p>	 <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p>	<p><math>4 + 7 + 6 = 10 + 7</math></p> <p><math>= 17</math></p> <p>Combine the two numbers that make 10 and then add on the remainder.</p>
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### Stage 5

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

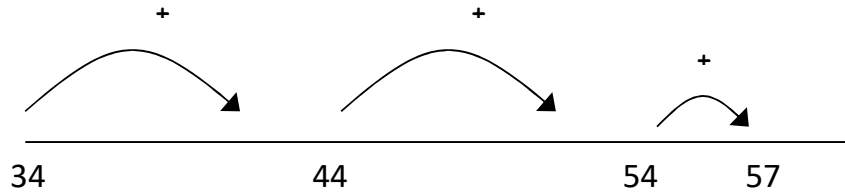
✓ First counting on in tens and ones.

$34 + 23 = 57$



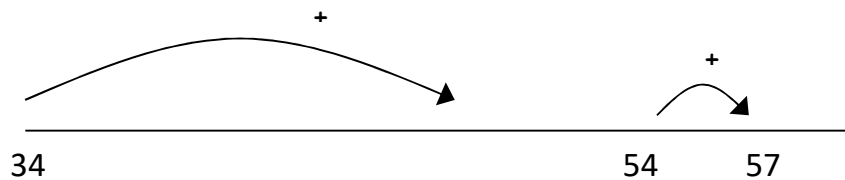
- ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact  $4 + 3 = 7$ ).

$$34 + 23 = 57$$



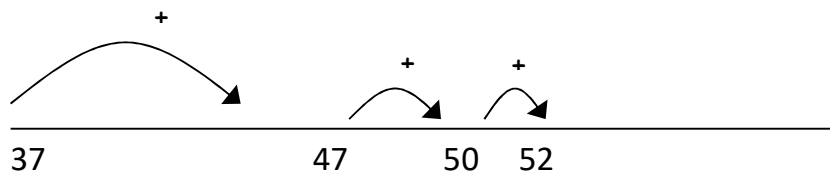
- ✓ Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$



- ✓ Bridging through ten can help children become more efficient.

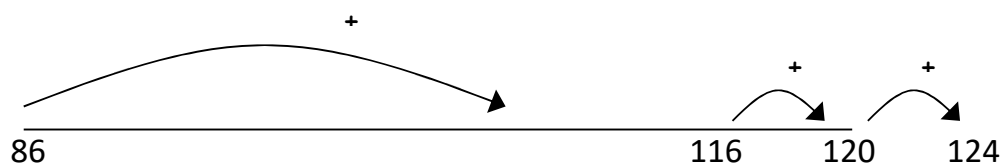
$$37 + 15 = 52$$



Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

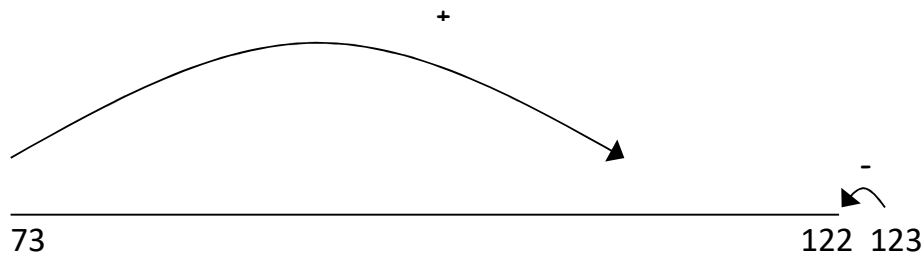
- ✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



✓ Compensation

$$49 + 73 = 122$$



Children will continue to use jottings to support, record and explain partial mental methods building on existing mental strategies.

✓ Record steps in addition using partitioning:

$$47 + 76 = 47 + 70 + 6 = 117 + 6 = 123$$

$$47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123$$

✓ Partitioned numbers are then written under one another:

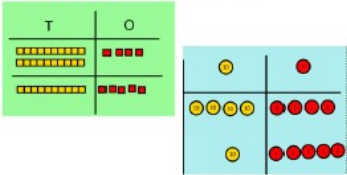
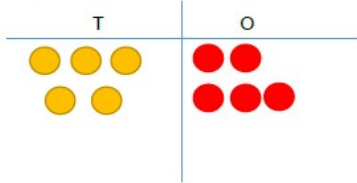
$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +76 & & +70 + 6 \\ \hline & & 110 + 13 = 123 \end{array}$$

✓ Recording is reduced further to:

$$\begin{array}{r} 47 \\ + 76 \\ \hline 13 \\ \hline 110 \\ \hline 123 \end{array}$$

This step moves to adding the least significant digits first in preparation for 'carrying.' This use of partition and expanded column addition is to be supported by the methods below.

## Concrete/Pictorial/Abstract:

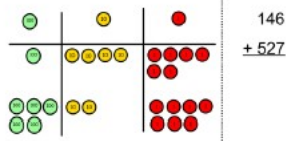
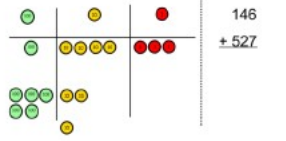
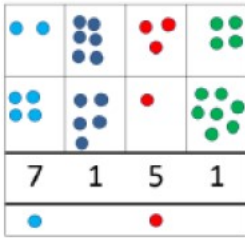
<p><math>24 + 15 =</math> Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p> 	<p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p> 	<p><u>Calculations</u></p> $21 + 42 =$ $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$
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## Stage 6

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred' **not** 'carry one.'

$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ 11 \end{array}$	$\begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array}$	$\begin{array}{r} 366 \\ + 458 \\ \hline 824 \\ 11 \end{array}$
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## Concrete/Pictorial/Abstract:

<p>Make both numbers on a place value grid.</p>  <p><math>146 + 527</math></p> <p>Add up the units and exchange 10 ones for one 10.</p>  <p>Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.</p> <p>This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.</p> <p>As children move on to decimals, money and decimal place value counters can be used to support learning.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p> 	<p>Start by partitioning the numbers before moving on to clearly show the exchange below the addition.</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.</p> $\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$ <p><math>11</math></p> $\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$
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*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ 1 \ 1 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ 1 \ 1 \ 1 \end{array}$$

*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ 1 \ 1 \ 1 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ 1 \ 1 \ 1 \end{array}$$

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ + 4681 \\ \hline 11944 \\ 1 \ 2 \ 1 \end{array}$$

*Using similar methods, children will*

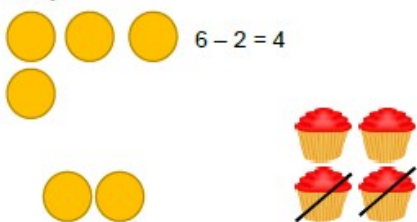
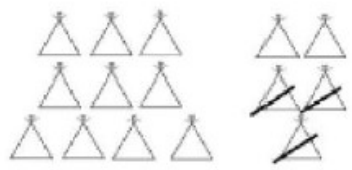
- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.*



## Subtraction

### Stage 1

- Lots of practical activities and counting rhymes and songs;
- Teacher models written recording using objects, marks and pictures;
- Children begin to make own marks during structured play.

Concrete	Pictorial
<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>6 - 2 = 4</p> <p>The concrete representation shows 6 yellow circles arranged in two rows of three. To the right, there are 4 red cupcake counters, with 2 of them crossed out with a diagonal line.</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>15 - 3 = 12</p> <p>The pictorial representation shows 15 triangles arranged in three rows of five. To the right, 3 triangles are crossed out with diagonal lines. Below the triangles, the equation 15 - 3 = 12 is written, with the answer 12 in a box.</p>

### Stage 2

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

### Example



They can record using marks and striking through them individually or as a group as they subtract.

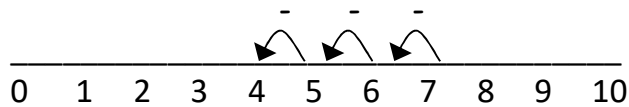
Example

$$10 - 6 = 4$$

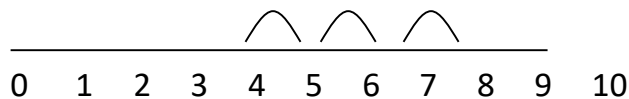
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They use numberlines and practical resources to support calculation. Teachers demonstrate the use of the numberline.

$$6 - 3 = 3$$

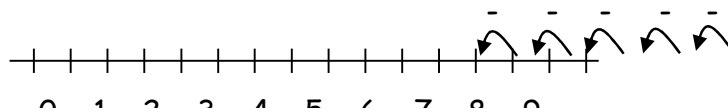


The numberline should also be used to show that  $6 - 3$  means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



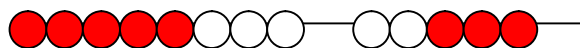
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

$$13 - 5 = 8$$



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.



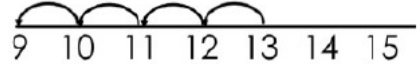
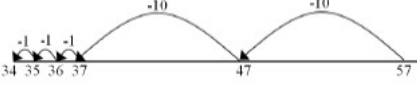
$$13 - 5 = 8$$



By the end of stage 2 children are selecting their own apparatus, using jottings (informal pencil and paper methods) and recording number sentences.

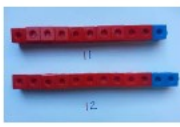
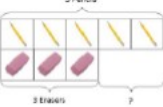
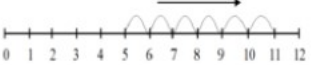
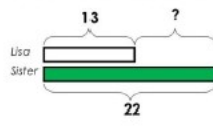
They understand that difference is subtraction.

Concrete/Pictorial/Abstract:

<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>13 - 4</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>
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Finding the difference:

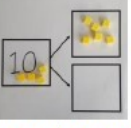
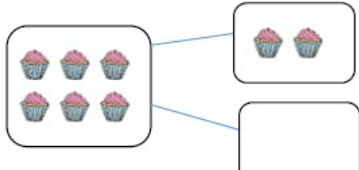
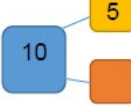
Concrete/Pictorial/Abstract:

<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference</p>	<p>Count on to find the difference.</p>  <p>Comparison Bar Models</p> <p>Draw bars to find the difference between 2 numbers.</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p> 	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
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### Stage 3

Children are encouraged to look at subtraction problems as a part, part, whole model.



Concrete/Pictorial/Abstract:

<p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction.</p>  <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>10 - 6 =</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
--	--	---

### Stage 4:

Children are encouraged to make 10 first when subtracting larger numbers.

Concrete/Pictorial/Abstract:

<p><math>14 - 9 =</math></p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.</p>	<p><math>13 - 7 = 6</math></p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p><math>16 - 8 =</math></p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p>
--	--	--

Children are encouraged to use a number square to subtract by counting up or down, initially in units, then in tens and units.

#### Example 1

$15 - 8 = 7$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

#### Example 2

$78 - 35 = 43$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

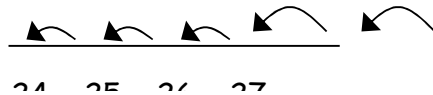
It is important to change the format of the square to that children become aware that the positions of numbers can change. Children will begin to use empty number lines to support calculations. They will continue to use jottings and practical resources, selecting for themselves which best suits their needs or with support and guidance where required.

### Stage 5

#### Counting back using “empty number lines”

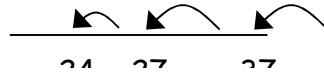
- ✓ First counting back in tens and ones.

$$47 - 23 = 24$$



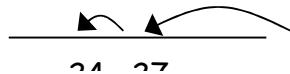
- ✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact  $7 - 3 = 4$ ).

$$47 - 23 = 24$$



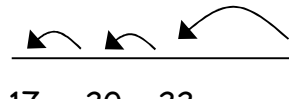
- ✓ Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



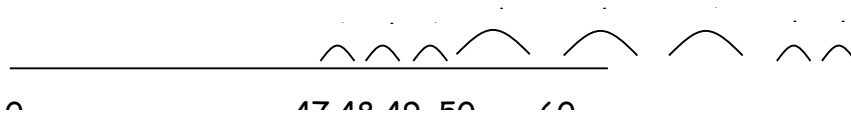
### Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so they then associate this method with 'taking away'.

$$82 - 47$$



### Help children to become more efficient with counting on by:

- ✓ Subtracting the units in one jump;
- ✓ Subtracting the tens in one jump and the units in one jump;
- ✓ Bridging through ten.

Children will continue to use empty number lines with increasingly large numbers. They will use jottings to support, record and explain partial mental methods building on existing mental strategies.

### Stage 6

#### Partitioning and decomposition

Children will learn to use more formal pencil and paper methods beginning with partitioning the numbers into tens and units and writing one under the other, initially using amounts that do not need the children to exchange.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 + 9 \\ 50 + 7 \\ \hline 30 + 2 \end{array} = 32$$

The expanded method leads children to the more compact column method so that they understand its structure and efficiency.

Concrete/Pictorial/Abstract:

<p>Use Base 10 to make the bigger number then take the smaller number away.</p> <p>Show how you partition numbers to subtract. Again make the larger number first.</p>	<p>Draw the Base 10 or place value counters alongside the written calculation to help to show working.</p> <p>Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>Calculations</p> $\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$	<p>This will lead to a clear written column subtraction.</p> $\begin{array}{r} 47 - 24 = 23 \\ \begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array} \end{array}$
--	--	--

**From this the children will begin to exchange.**

This would be recorded by the children as

$$\begin{array}{r} 71 \\ - 46 \\ \hline \end{array} \quad \begin{array}{r} 60 \\ \cancel{70} + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

**Partitioning and decomposition**

$$\begin{array}{r} 754 \\ - 86 \\ \hline \end{array}$$

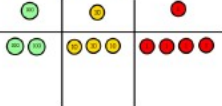
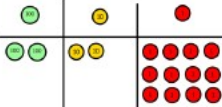
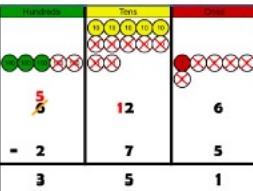

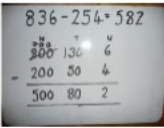
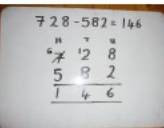
This would be recorded by the children as

$$\begin{array}{r} 600 \quad 140 \\ \cancel{700} + \cancel{50} + 14 \\ - \quad \quad 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$$

The expanded method leads children to the more compact column method so that they understand its structure and efficiency.

$$\begin{array}{r} 6141 \\ 754 \\ - 86 \\ \hline 668 \end{array}$$

Concrete/Pictorial/Abstract:

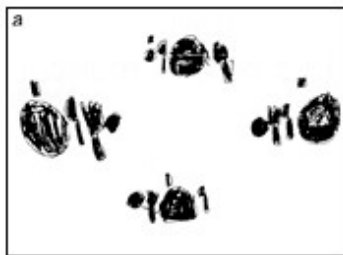
<p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> <p>Make the larger number with the place value counters</p>  <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ <p>Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p>  <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$	 <p>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.</p>  <p>When confident, children can find their own way to record the exchange/regrouping.</p> <p>Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.</p>	 <p>Children can start their formal written method by partitioning the number into clear place value columns.</p>  <p>Moving forward the children use a more compact method.</p>
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**Multiplication**

**Stage 1**

Children will experience equal groups of objects and will count in 2s, 5s and 10s. They will work on practical problem solving activities involving equal sets or groups.





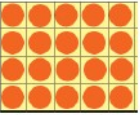



Children will develop their understanding of multiplication and use jottings and pictures to support calculations.

**Arrays**

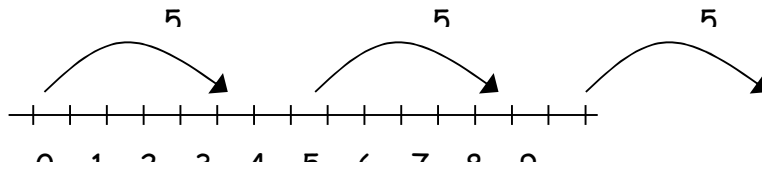
Children will use arrays to model a multiplication. This knowledge will support with the development of the grid method later.

○ ○ ○ ○ ○  
○ ○ ○ ○ ○      $5 \times 3 = 15$   
○ ○ ○ ○ ○  
  
 $3 \times 5 = 15$

Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find commutative multiplication sentences.  $4 \times 2 = 8$ $2 \times 4 = 8$  $2 \times 4 = 8$ $4 \times 2 = 8$  Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
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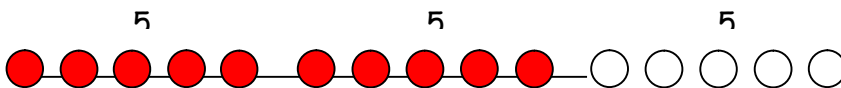
Children will use repeated addition to develop their understanding of multiplication. This can be shown easily on a number line:

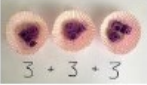




$$5 \times 3 = 5 + 5 + 5$$



And on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$

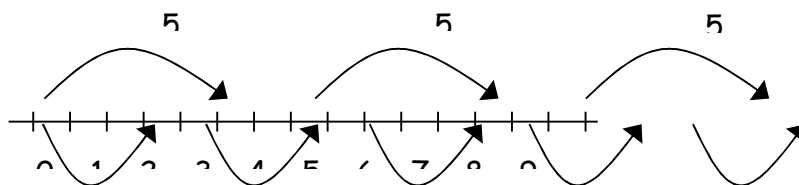


<p>Repeated addition</p>	 $2 + 2 + 2$  <p>Use different objects to add equal groups.</p>	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>  <p>2 add 2 add 2 equals 6</p>  <p><math>5 + 5 + 5 = 15</math></p>	<p>Write addition sentences to describe objects and pictures.</p>  <p><math>2 + 2 + 2 + 2 = 10</math></p>
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## Stage 2

### Commutativity

Children should know that  $3 \times 5$  has the same answer as  $5 \times 3$ . This can also be shown on the number line.



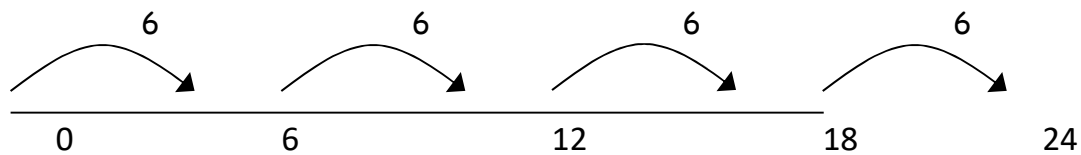
3                      3                      3                      3                      3

Children will be introduced to inverse operations and come to understand that  $3 \times 5 = 15$  or  $5 \times 3 = 15$  can be checked by using division  $15 \div 3 = 5$  or  $15 \div 5 = 3$ .

### Stage 3

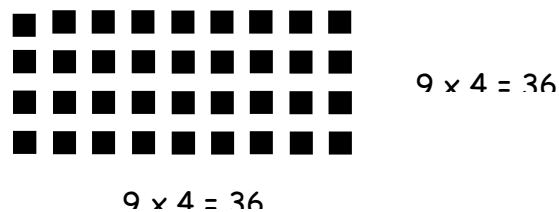
Children will continue to use repeated addition and use number lines to support their understanding.

4 times 6 is  $6 + 6 + 6 + 6 = 24$  or 4 lots of 6 or  $6 \times 4$



### Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



Children will also develop an understanding of:

### Scaling

e.g. Find a ribbon that is 4 times as long as the blue ribbon



5 cm

20 cm

**Using symbols**

Unknown numbers are represented by a symbol for children to complete equations using inverse operations.

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \bigcirc = 32$$

**Partitioning**

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

Stage 5

**Grid Method**

**TU x U** (short multiplication by a single digit)

Children will approximate first,  $38 \times 7$  is approximately  $40 \times 7 = 280$

x	30	8	
7	210	56	= 266

### Grid Method

Show the link with arrays to first introduce the grid method.

4 rows of 10  
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.

Calculations  
4 x 126

Fill each row with 126.

Calculations  
4 x 126

Add up each column, starting with the ones making any exchanges needed.

Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

$210 + 35 = 245$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

## Tower Method

$$\begin{array}{r}
 30 + 8 \\
 \times \quad 7 \\
 \hline
 56 \\
 210 \\
 \hline
 266
 \end{array}$$

$$\begin{array}{r}
 38 \\
 \times 7 \\
 \hline
 56 \\
 210 \\
 \hline
 266
 \end{array}$$

## Compact Method

$$\begin{array}{r}
 38 \\
 \times 7 \\
 \hline
 266 \\
 5
 \end{array}$$

Remind children to start at the units. The method is starting at  $7 \times 8 = 56$ . Put 6 in the box and remember the 50; to help you, put 5 tens under the tens column. Then  $7 \times 30$  makes 210; add on the 50 which equals 260 – put 26 in the box under the correct columns.

The above three methods can then be extended to use with bigger numbers.

$$56 \times 27$$

TU x TU

x	50	6
20	1000	120
7	350	42

$$1350 + 162 = \underline{1512}$$

Encourage jottings. Approximate first.  $56 \times 27$  is approximately  $60 \times 30 = 1800$

56		
<u>X 27</u>		
392	$7 \times 56 \longrightarrow$	56
<u>1120</u>	$30 \times 56$	<u>x 7</u>
<u>1512</u>		<u>392</u>
1		4

HTU x TU

Teacher demos only. Discuss problems this method can cause.

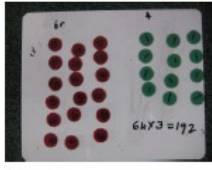
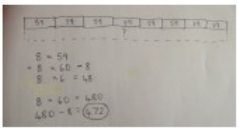
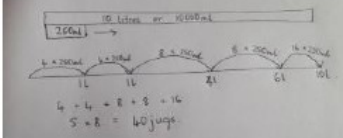
x	200	80	6	
20	4000	1600	120	5720
9	1800	720	54	2574
				<u>8294</u>

Encourage jottings if necessary.

286 x 29 is approximately 300 x 30 = 9000

$$\begin{array}{r} 286 \\ \times 29 \\ \hline 2574 \\ 5720 \\ \hline 8294 \\ 1 \end{array}$$

$$\begin{array}{r} 286 \\ \times 9 \\ \hline 2574 \\ 75 \end{array}$$

<p><b>Column multiplication</b></p>	<p>Children can continue to be supported by place value counters at the stage of multiplication.</p>  <p>It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.</p>	<p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>  	<p>Start with long multiplication, reminding the children about lining up their numbers clearly in columns.</p> <p>If it helps, children can write out what they are solving next to their answer.</p> $\begin{array}{r} 32 \\ \times 24 \\ \hline 128 \\ 640 \\ \hline 768 \end{array}$ <p>(4 x 2) (4 x 30) (20 x 2) (20 x 30)</p> <p>This moves to the more compact method.</p> $\begin{array}{r} 32 \\ \times 24 \\ \hline 128 \\ 640 \\ \hline 768 \end{array}$
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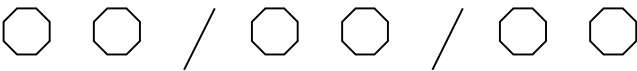
## Division:

### Stage 1

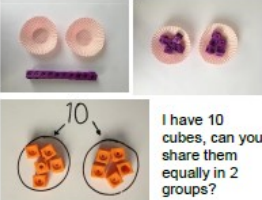

Children will understand equal groups and share items out in play and problem solving. They will count in 2s, 5s and 10s.



There are 6 sweets, how many people can have 2 sweets each?



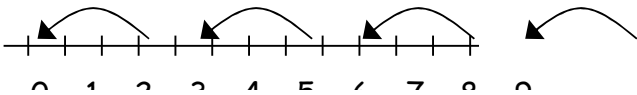
Recording will be linked to pictures  $6 \div 3 = 2$

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p><math>8 \div 2 = 4</math></p>	<p>Share 9 buns between three people.</p> <p><math>9 \div 3 = 3</math></p>

**Stage 2**

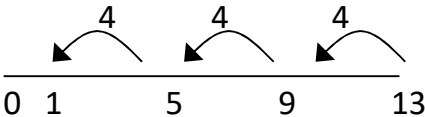
Children will use repeated subtraction to develop their understanding using a number line.

$12 \div 3 = 4$

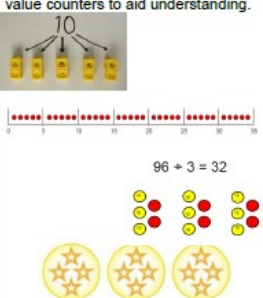
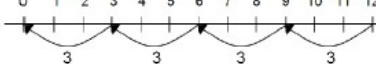
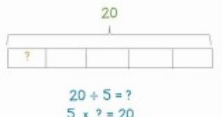


Children will be introduced to calculations involving remainders

$13 \div 4 = 3 \text{ r } 1$





<p><b>Division as grouping</b></p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>96 ÷ 3 = 32</p>	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p>20 ÷ 5 = ? 5 x ? = 20</p>	<p>28 ÷ 7 = 4</p> <p>Divide 28 into 7 groups. How many are in each group?</p>
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### Using symbols

Unknown numbers are represented by a symbol for children to complete equations using inverse operations.

$$\square \div 2 = 4$$

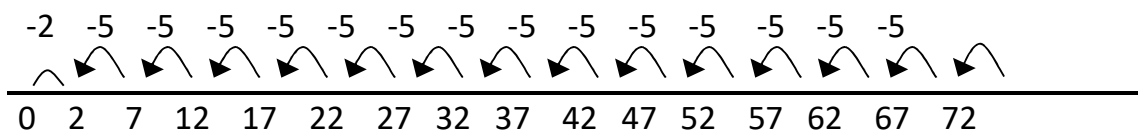
$$20 \div \triangle = 4$$

$$\square \div \triangle = 4$$

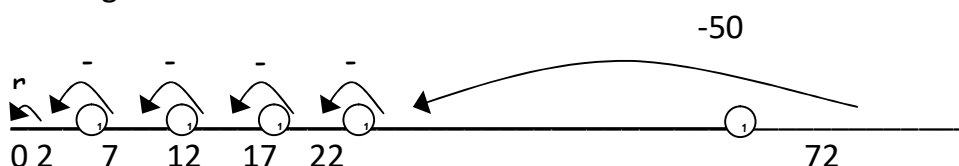
### Stage 3

Children will continue to use number lines and repeated subtraction but the emphasis is on grouping rather than sharing. Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

$$72 \div 5$$



Moving onto:




## Stage 4

### Vertical Method

#### Short division TU $\div$ U


$$72 \div 3$$

$$\begin{array}{r} 24 \\ 3 \overline{) 72} \\ - 30 \quad 10x \\ \hline 42 \\ - 30 \quad 10x \\ \hline 12 \\ - 6 \quad 2x \\ \hline 6 \\ - 6 \quad 2x \\ \hline 0 \end{array}$$


Answer: 24

Leading to subtraction of other multiples.

$$96 \div 6$$

$$\begin{array}{r} 16 \\ 6 \overline{) 96} \\ - 60 \quad 10x \\ \hline 36 \\ - 36 \quad 6x \\ \hline 0 \end{array}$$


Answer: 16

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

$$97 \div 9$$

$$\begin{array}{r} \underline{10 \text{ r}7} \\ 9 \overline{) 97} \\ \underline{90} \quad 10 \times 9 \\ 7 \end{array}$$

As you record the division ask, 'How many nines in 90?' or 'What is 90 divided by 9?' The '7' left at the bottom must be highlighted as the remainder as '9' cannot go into '7'.

When children know the multiplication facts with ready recall, this can be shortened.

$$\begin{array}{r} \underline{2 \ 7} \\ 3 \overline{) 8 \text{ } ^2 1} \end{array}$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. This method can then be used for remainders.

### **Problem Solving**

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example  $62 \div 8$  is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box)

<p><b>Short division</b></p>	<p>Tens      Units</p> <p>3      2</p> <p>Use place value counters to divide using the bus stop method alongside</p> <p>42 ÷ 3 =</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p> <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p> <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p> <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 258} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$
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## Stage 5

When dealing with  $\text{HTU} \div \text{U}$ , the method often referred to as 'chunking' is based on subtracting multiples of the divisor, or 'chunks.' Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.

$$196 \div 6$$

$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ \underline{60} \\ 136 \\ \underline{60} \\ 76 \\ \underline{60} \\ 16 \\ \underline{12} \\ 4 \end{array}$	$\begin{array}{l} 10 \times 6 \\ 10 \times 6 \\ 10 \times 6 \\ 2 \times 6 \end{array}$	<p>and</p> $\begin{array}{r} 30 + 2 \text{ r } 4 \\ 6 \overline{) 180 + 16} \\ \hline = 32 \text{ r } 4 \end{array}$
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Chunking is useful for reminding children of the link between division and repeated subtraction.

Estimating has two purposes when doing a division:

- to help to choose a starting point for the division;

- to check the answer after the calculation.

$$196 \div 6$$

$$\begin{array}{r} \underline{32 \text{ r}4} \\ 6 \overline{) 196} \\ \underline{180} \phantom{00} \quad 30 \times 6 \\ 16 \phantom{00} \\ \underline{12} \phantom{00} \\ 4 \phantom{00} \end{array}$$

Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally and whose understanding of partitioning and place value is sound.

$$291 \div 3$$

$$\begin{array}{r} \underline{97} \\ 3 \overline{) 291} \end{array}$$

$$\text{HTU} \div \text{TU}$$

$$\begin{array}{r} \underline{23 \text{r}8} \\ 24 \overline{) 560} \\ - \underline{480} \phantom{00} \quad 20 \times 24 \\ 80 \phantom{00} \\ - \underline{72} \phantom{00} \quad 3 \times 24 \\ 8 \phantom{00} \end{array}$$

Answer = 23 r 8

This eventually leads to:

$$\begin{array}{r} \underline{23 \text{r}8} \\ 24 \overline{) 560} \\ - \underline{480} \\ 80 \end{array}$$

-  $\frac{72}{8}$

Answer = 23 r 8

Long division

2544 ÷ 12

How many groups of 12 thousands do we have?  
None

Exchange 2 thousand for 20 hundreds.

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.  
We have grouped 24 hundreds so can take them off and we are left with one.

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

0 3 1 8 r 5

20 | 6 3 6 5

6 0

3 6

2 0

1 6 5

1 6 0

5

The governing body reviewed this policy on Monday,25<sup>th</sup> March 2024

Review Date – March 2025

Signed \_\_\_\_\_

Headteacher

Signed \_\_\_\_\_

Chair of Governors